Metal Forming – BSc 2024/25-1

Calculation methods

Introduction

Full solutions:

Establish a precise physical model for the given forming task and apply a precise mathematical solution.

These lead to a partial differential equation system.

Generally, these equation systems can't be solved.

Therefore, simplifications are applied to the physical model or the mathematical solution, or even on both.

The simplified solutions are made for the two border strain case: axial symmetrical and plain strain.

Introduction

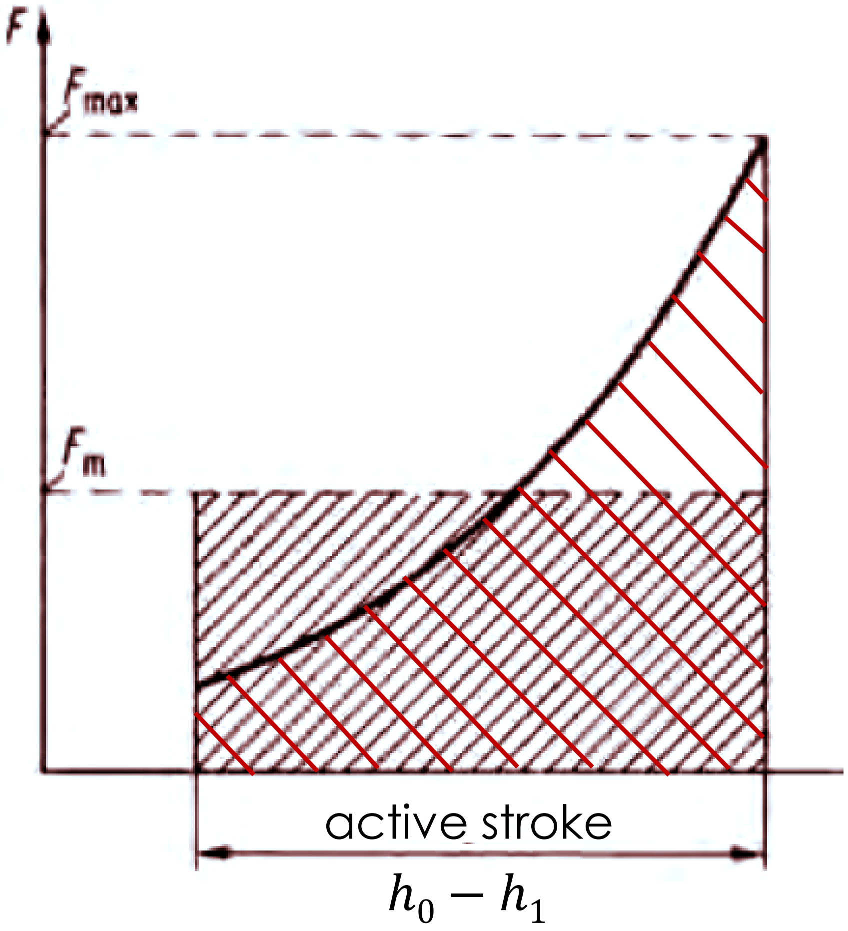
Fast estimation (for upsetting):

$$W = \frac{V \cdot \sigma_{fm} \cdot \varphi_{eq}}{\eta_F}$$

or

$$W = F(h_0 - h_1) \cdot x$$

$$x = \frac{F_m}{F_{max}} \qquad x \cong 0.6$$



- W - upsetting work
- V - volume involved in deformation
- σ_{fm} - mean flow stress
- φ_{eq} - equivalent strain
- η_F - deformation efficiency (0.6 – 0.9)
- h_0 - stock height
- x - process factor
- F_m - mean force
- F_{max} - maximum force

Border cases

Simplified solutions (in principal coordinate system):

Axial symmetrical case

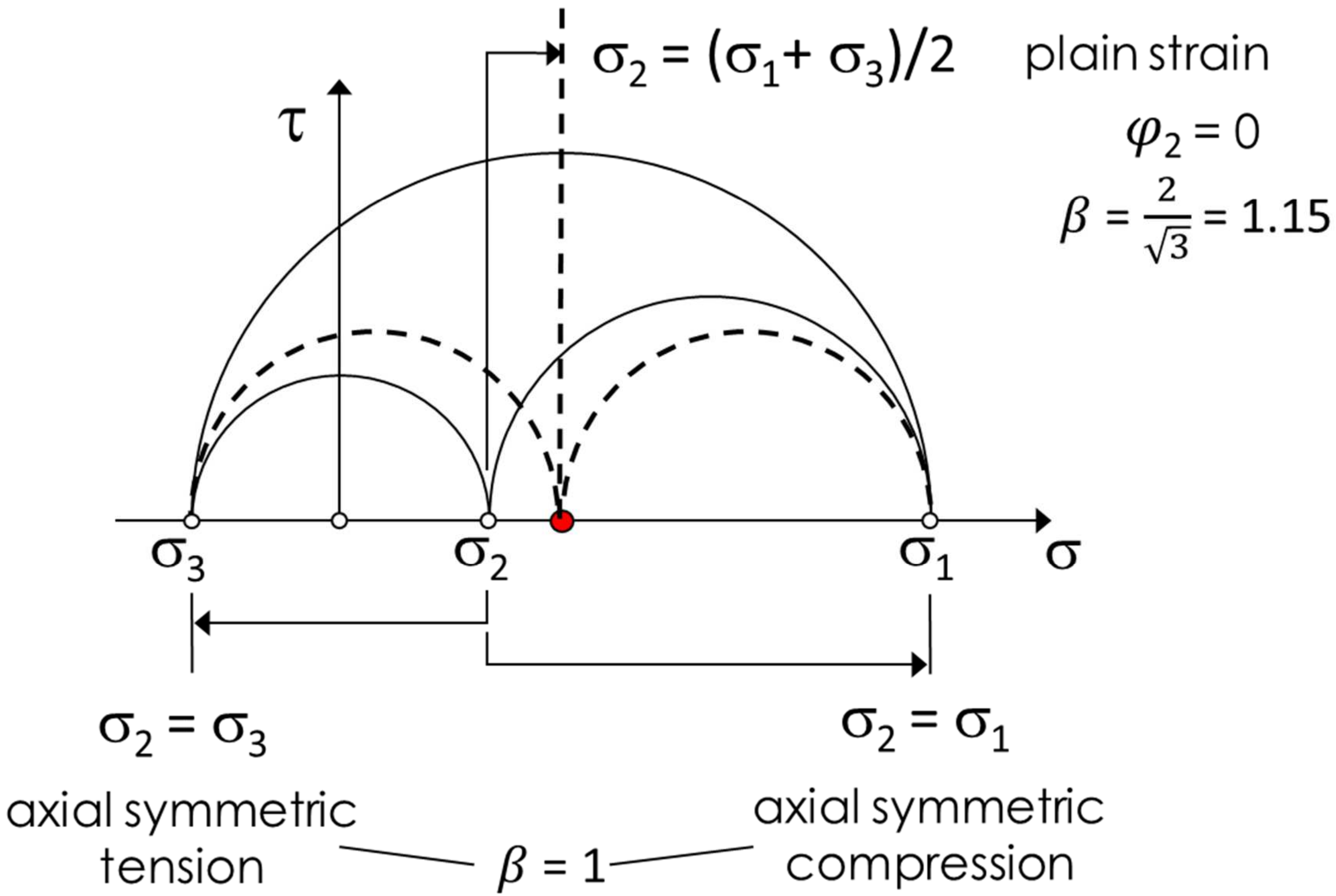
The strains and also the stresses, which are perpendicular to the axis, are identical.

Plain strain

In the second main direction, the strain is zero. To ensure this, the second main stress can not be zero.

Border cases

General flow criterion: $\sigma_1 - \sigma_3 = \beta\, \sigma_f$



Main methods

Solution technics

- Equilibrium Approach
- Energy Approach
- Others …

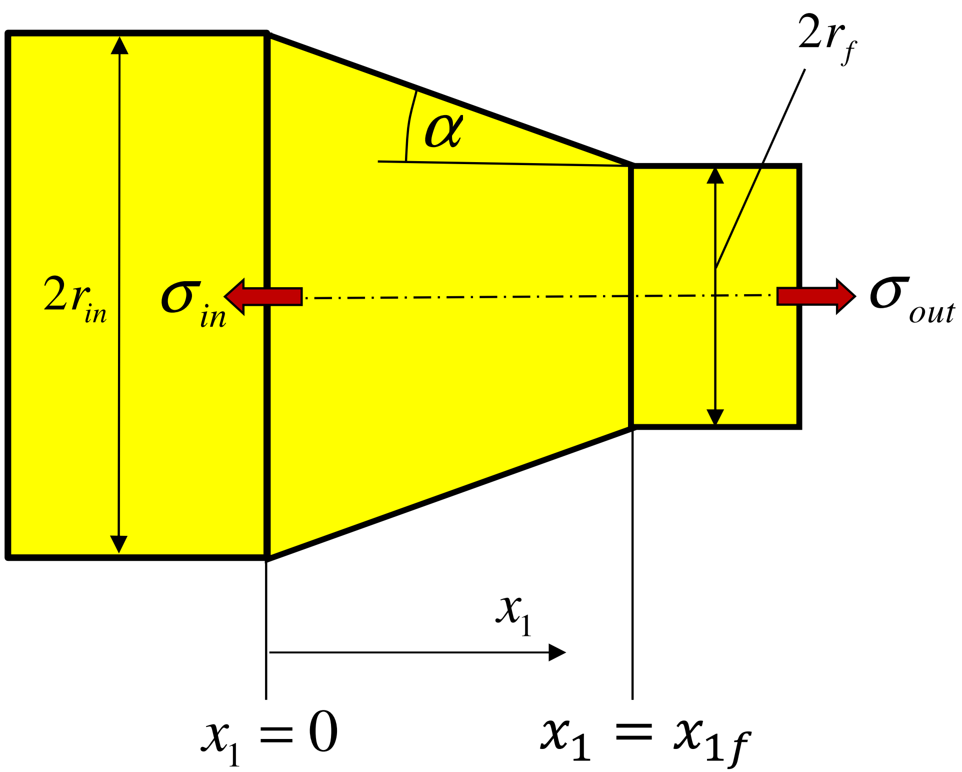
Equilibrium Approach

Supposed conditions

- 1D stress and strain distribution
- Homogeneous fields perpendicular to the direction of change (flow)
- Simplified flow conditions
- Simplified geometry
- Approximate boundary conditions

Axial symmetric case

Material flow in conical dies



Drawing $(\sigma_{out} < \sigma_{f\,out})$

$x_1 = 0, \quad \sigma_{11} = \sigma_{in} = 0$

$x_1 = x_{1f}, \quad \sigma_{11} = \sigma_{out} < \sigma_{fout}$

Reduction $(\sigma_{in} < \sigma_{fin})$

$x_1 = x_{1f}, \quad \sigma_{11} = \sigma_{out} = 0$

$x_1 = 0, \quad \sigma_{11} = \sigma_{in} < \sigma_{fin}$

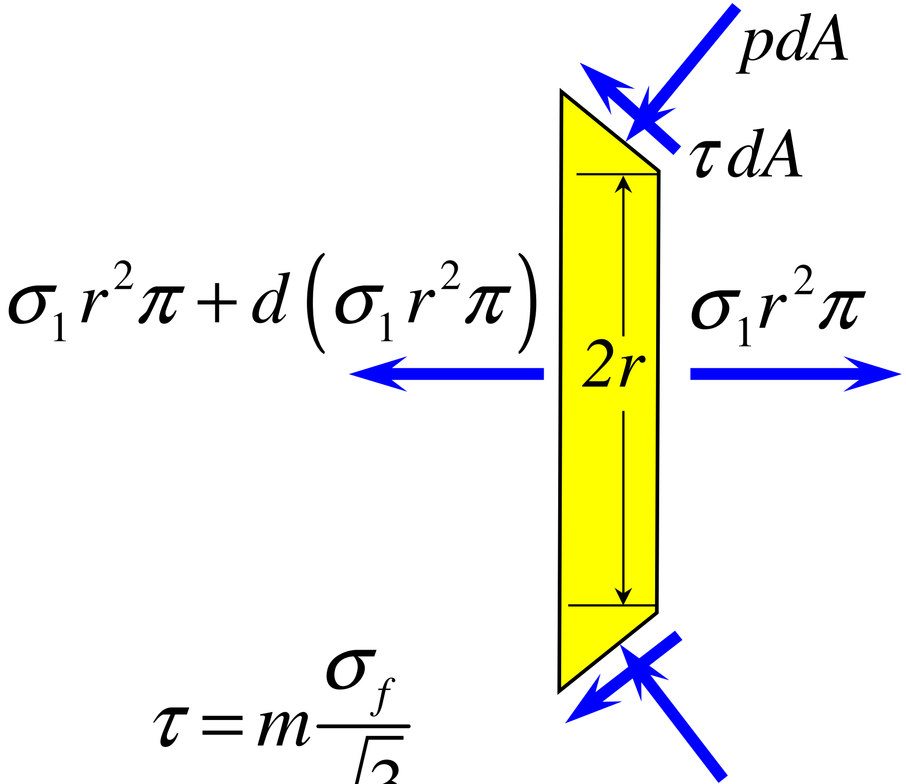
Extrusion

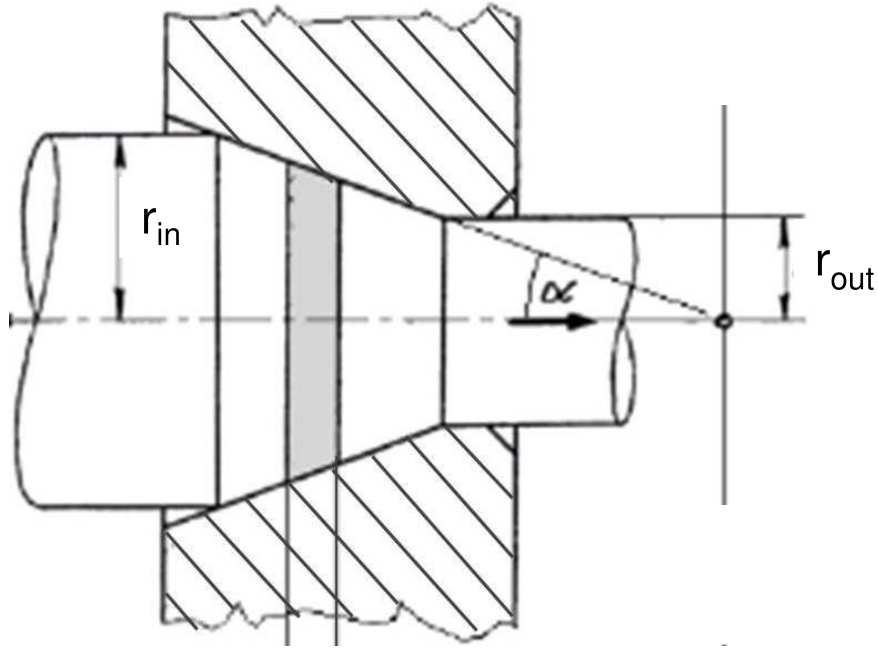
$x_1 = 0, \quad \sigma_{11} = \sigma_{in} + \sigma_{w\,fr} > 0$

$x_1 = x_{1f}, \quad \sigma_{11} = \sigma_{out} = 0$

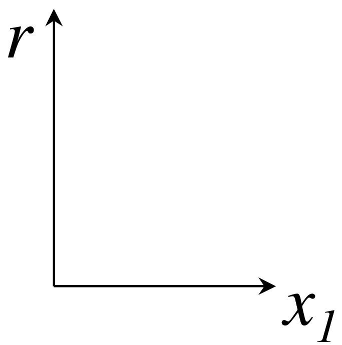
$\sigma_{w\,fr}$ is additional stress from the friction with the recipient wall.)

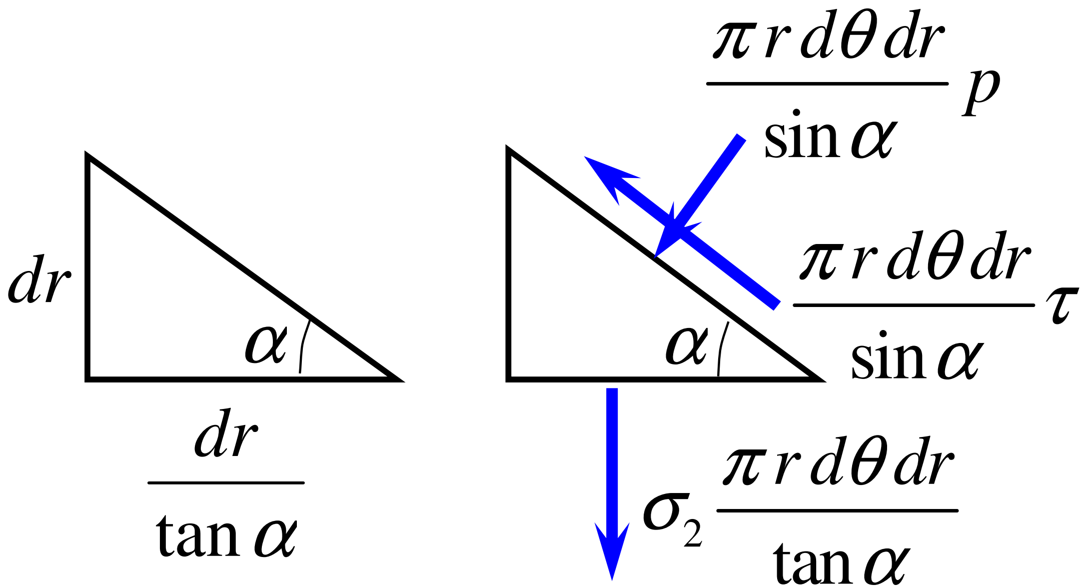
Axial symmetric case





$$\tau = m\frac{\sigma_f}{\sqrt{3}}$$





Axial symmetric case

$$\sum F_i = 0$$

$$\sum F_{x_1} = -\,d(\sigma_1 r^2 \pi) - 2\,\tau\,\pi\,r\,dr\,\cot\alpha - 2\,p\,\pi\,r = 0$$

$$d(\sigma_1 r^2) + 2(\tau \cot\alpha + p) r\,dr = 0$$

$$d(\sigma_1 r^2) + 2\left(m\frac{\sigma_f}{\sqrt{3}} \cot\alpha + p\right) r\,dr = 0$$

$$\sum F_r = \tau\,r\,dr\,d\theta - p\,r\,dr\,d\theta\,\cot\alpha - \sigma_r\,r\,dr\,d\theta\,\cot\alpha$$

$$p = \tau \tan\alpha - \sigma_r = m\frac{\sigma_y}{\sqrt{3}} \tan\alpha - \sigma_r$$

$$\sigma_1 - \sigma_r = \sigma_f \qquad \frac{d\sigma_1}{4\sigma_1 + H\sigma_f} = \frac{dr}{r}$$

$$H = m\sqrt{3}\left(\cot\alpha - \tan\alpha - \sqrt{3}\right) \qquad r = r_{in},\ \ \sigma_1 = 0$$

$$\sigma_1 = \frac{H}{4}\sigma_f\left(\left(\frac{r}{r_{be}}\right)^4 - 1\right) \qquad \sigma_r = \frac{H}{4}\sigma_f\left(\left(\frac{r}{r_{be}}\right)^4 - 1\right) - \sigma_f$$

$$\sigma_f = C_1 + C_2\bar{\varphi}^n \qquad \bar{\varphi} = 2\ln\frac{r_{in}}{r}$$

Axisymmetric case

Coulomb friction

$$\tau = \mu p$$

$$r d\sigma_1 + 2\sigma_1 dr + 2 p dr(1 + \mu \cot\alpha) = 0$$

$$p = \tau \tan\alpha - \sigma_r = p\mu \tan\alpha - \sigma_r \rightarrow p(1 - \mu \tan\alpha) = -\sigma_r$$

$$\sigma_1 - \sigma_r = \sigma_Y$$

$$\frac{d\sigma_1}{\sigma_1 B - \sigma_Y(1+B)} = \frac{2dr}{r}$$

$$\sigma_1 = \frac{Cr^{2B}}{B} + \frac{1+B}{B}\sigma_Y, \; r = r_x, \; \sigma_1 = \sigma_{1x}$$

$$\frac{\sigma_1}{\sigma_Y} = \frac{1+B}{B}\left[1 - \left(\frac{r}{r_x}\right)^{2B}\right] + \frac{\sigma_{1x}}{\sigma_Y}\left(\frac{r}{r_x}\right)^{2B}$$

$$1 + B = \left(1 + \frac{\mu}{\tan\alpha}\right)\frac{1}{1 - \mu\tan\alpha}$$

Average yield stress

For the solution of differential equations, we assume that the yield stress is constant during the process (by using the average value).

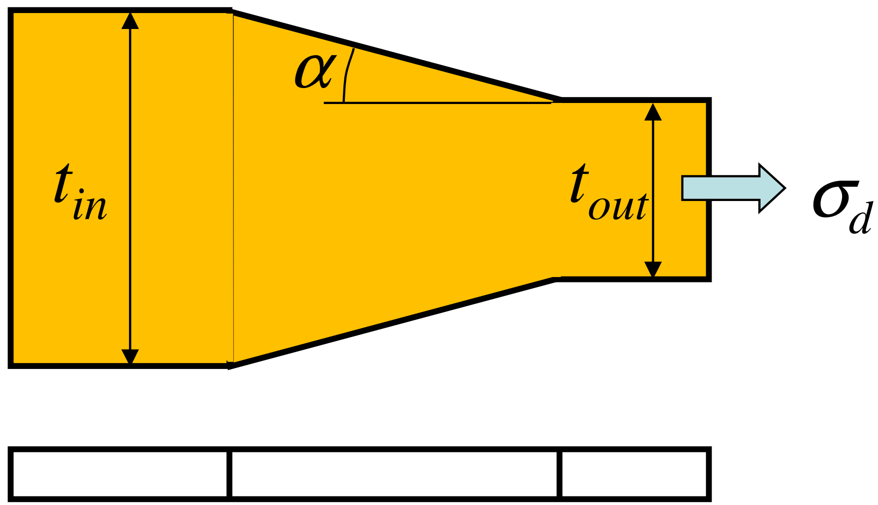
$$\sigma_{f_{average}} = \tilde{\sigma}_f = \frac{1}{\bar{\varphi}_{max} - \bar{\varphi}_{min}} \int_{\bar{\varphi}_{min}}^{\bar{\varphi}_{max}} \sigma_f d\bar{\varphi}$$

$$\sigma_f = C_1 + C_2\bar{\varphi}^n$$

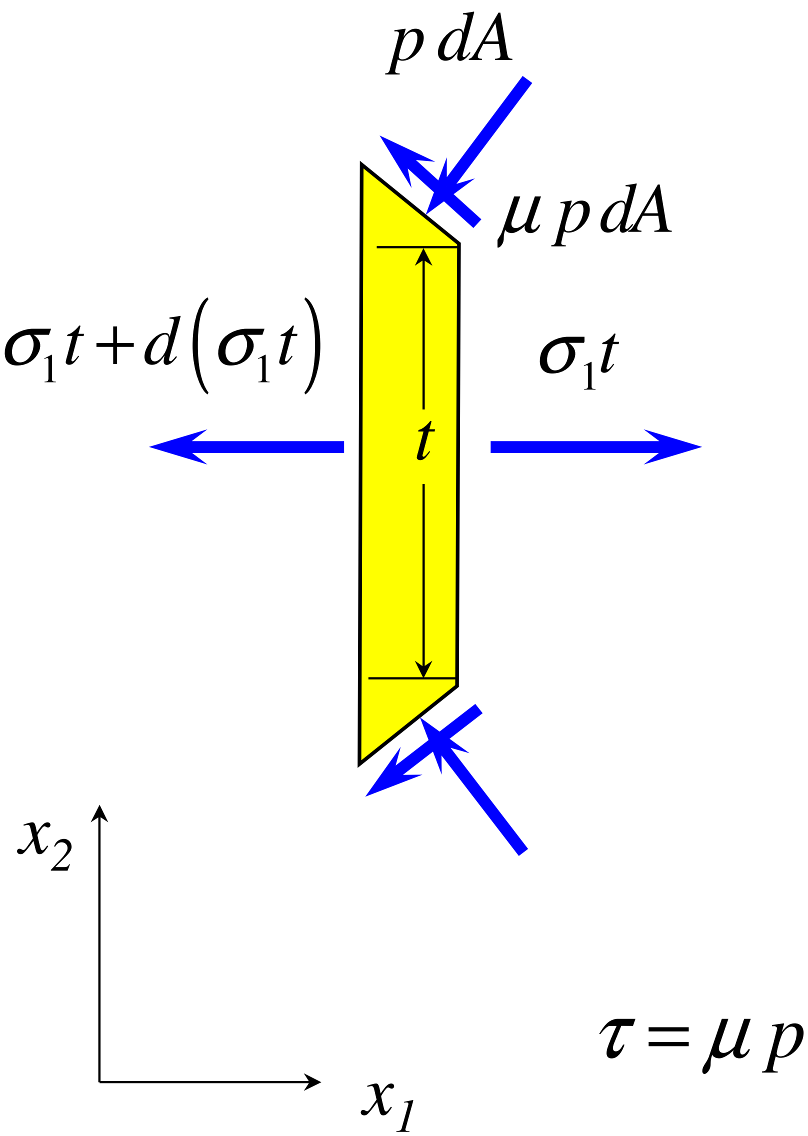
$$\tilde{\sigma}_f = \frac{1}{\bar{\varphi}_{max} - \bar{\varphi}_{min}} \left[C_1(\bar{\varphi}_{max} - \bar{\varphi}_{min}) + \frac{C_2}{n+1}\left(\bar{\varphi}_{max}^{n+1} - \bar{\varphi}_{min}^{n+1}\right) \right]$$

Flow trough a narrowing die

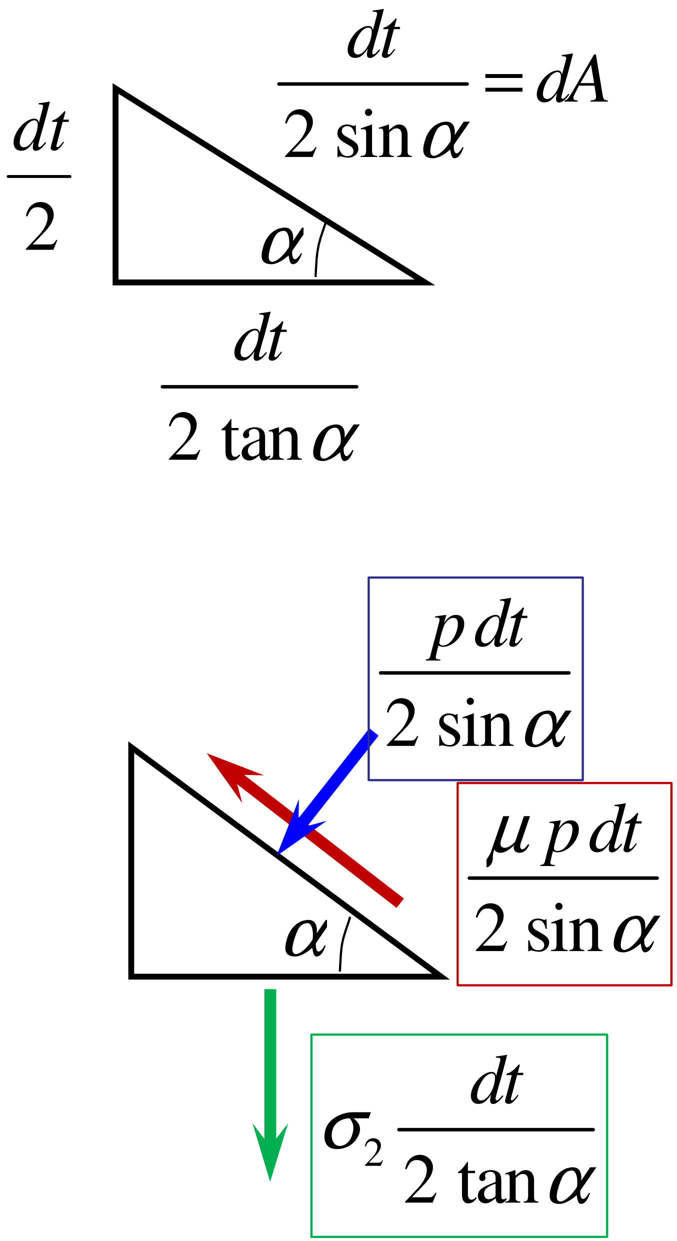
Plain strain case



Plain strain case



$$\tau = \mu\,p$$



Plain strain case

$$\sum F_i = 0$$

$$\sum F_{x_2} = 0 = \mu p \frac{dt}{2} - \frac{pdt}{2\tan\alpha} - \sigma_2 \frac{dt}{2\tan\alpha} = 0$$

$$\sigma_2 = p(\mu \tan\alpha - 1) \approx -p$$

$$\sum F_{x_1} = 0 = \sigma_1 t - \sigma_1 t - d(\sigma_1 t) - 2\frac{\mu pdt}{2\tan\alpha} - 2\frac{p}{2}dt = 0$$

$$\sigma_1 dt + td\sigma_1 = -pdt \cdot \left(\frac{\mu}{\tan\alpha} + 1\right) =$$

$$= -\sigma_2 dt\left(\frac{\mu}{\tan\alpha} + 1\right)(1 - \mu\tan\alpha) - \sigma_1 dt = \sigma_2 (B+1) dt$$

$$d\sigma_1 = (\sigma_2 (1+B) - \sigma_1)\frac{dt}{t}$$

$$B = \left(\frac{\mu}{\tan\alpha} + 1\right)(1 - \mu\tan\alpha) - 1$$

Plain strain case

Plain strain $\xi_2 = 0, \quad \xi_{ij} = \dot{\lambda}\sigma'_{ij} \;\rightarrow\; 0 = \dot{\lambda}\left(2\sigma_2 - \sigma_1 - \sigma_3\right)$

$$\sigma_2 = \frac{\sigma_1 + \sigma_3}{2},$$

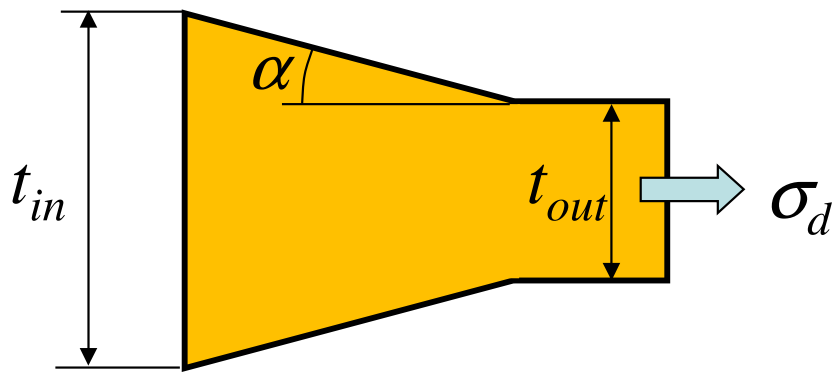
$$yield\ criteria: \;\; \bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_1 - \sigma_3\right)^2 + \left(\sigma_2 - \sigma_3\right)^2} = \sigma_f$$

$$\bar{\sigma} = \frac{\sqrt{3}}{2}\left|\sigma_1 - \sigma_3\right| = \frac{\sqrt{3}}{2}\left(\sigma_1 - \sigma_3\right),$$

$$\sigma_1 > \sigma_3, \quad \sigma_2 = \sigma_1 - \frac{2}{\sqrt{3}}\sigma_f$$

$$d\sigma_1 = \left[\left(1+B\right)\left(\sigma_1 - \frac{2}{\sqrt{3}}\sigma_y\right) - \sigma_1\right]\frac{dt}{t} \rightarrow \frac{d\sigma_1}{B\sigma_1 - \frac{2}{\sqrt{3}}\sigma_y\left(1+B\right)} = \frac{dt}{t}$$

Plain strain case



$$\frac{d\sigma_1}{B\sigma_1 - \frac{2}{\sqrt{3}}\sigma_f(1+B)} = \frac{dt}{t}$$

Boundary conditions: drawing

$$t = t_{in}, \quad \sigma_1 = 0$$

$$t = t_{out}, \quad \sigma_1 = \sigma_d$$

$$\int_{\sigma_1=0}^{\sigma_1=\sigma_d} \frac{d\sigma_1}{B\sigma_1 - \frac{2}{\sqrt{3}}\sigma_f(1+B)} = \int_{t=t_{in}}^{t=t_{out}} \frac{dt}{t} \rightarrow \sigma_d = \frac{2}{\sqrt{3}}\sigma_f \frac{(1+B)}{B}\left[1-\left(\frac{t_{out}}{t_{in}}\right)^B\right]$$

Plain strain case

$$\sigma_1(t) = \frac{2}{\sqrt{3}}\sigma_f \frac{(1+B)}{B}\left[1-\left(\frac{t}{t_{in}}\right)^B\right],$$

$$\sigma_2(t) = \frac{2}{\sqrt{3}}\sigma_f \frac{(1+B)}{B}\left[1-\left(\frac{t}{t_{in}}\right)^B\right] - \frac{2}{\sqrt{3}}\sigma_f$$

$$\sigma_f = \text{const.}$$

$$p = \frac{\sigma_2}{\mu \tan\alpha - 1}$$

$$\frac{d\sigma_1}{dt} - \left[(1+B)\left(\sigma_1(t) - \frac{2}{\sqrt{3}}\sigma_f(t)\right) - \sigma_1(t)\right]\frac{1}{t} = 0, \quad \bar{\varphi} = \frac{2}{\sqrt{3}}\ln\frac{t_{in}}{t}$$

Steps of equilibrium approach

- Choosing a direction characteristic to the deformation / stress.
- Defining the equilibrium of the forces acting on an elemental slice of the body, that is perpendicular to the chosen direction
- Construction of a differential equation (1D)
- Flow condition, reduction of the number of variables
- Solution of the DE with the boundary conditions

Energy Approach

Total power of outer forces

$$J = \int_V \sigma_f \bar{\xi} dV + \int_{A_\Gamma} \tau |\Delta v| \, dA_\Gamma - \int_{A_t} T_i \, v_i$$

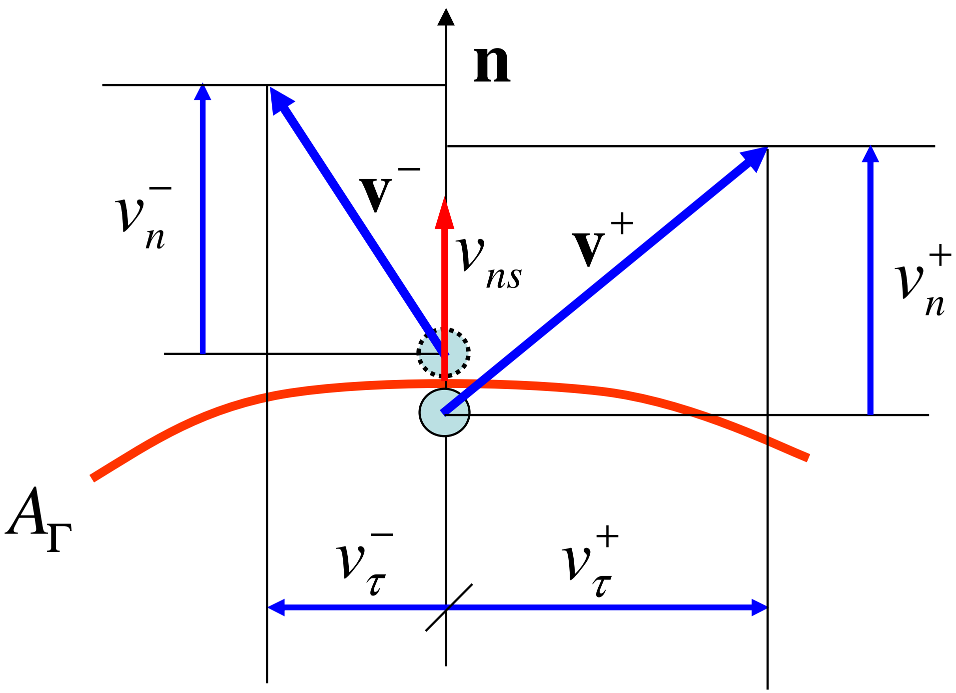
From the numerous kinematically possible velocity fields, the expression above is minimal for the actual (real) one.

First term: the power of inner forces
Second term: the power of discontinuity surfaces
Third term: the power of external constraints (e.g. wire drawing)

$$J = \dot{W}_i + \dot{W}_\Gamma - \dot{W}_t$$

Kinematic boundary conditions and incompressibility shall be valid for the real velocity field.

Discontinuity on boundaries

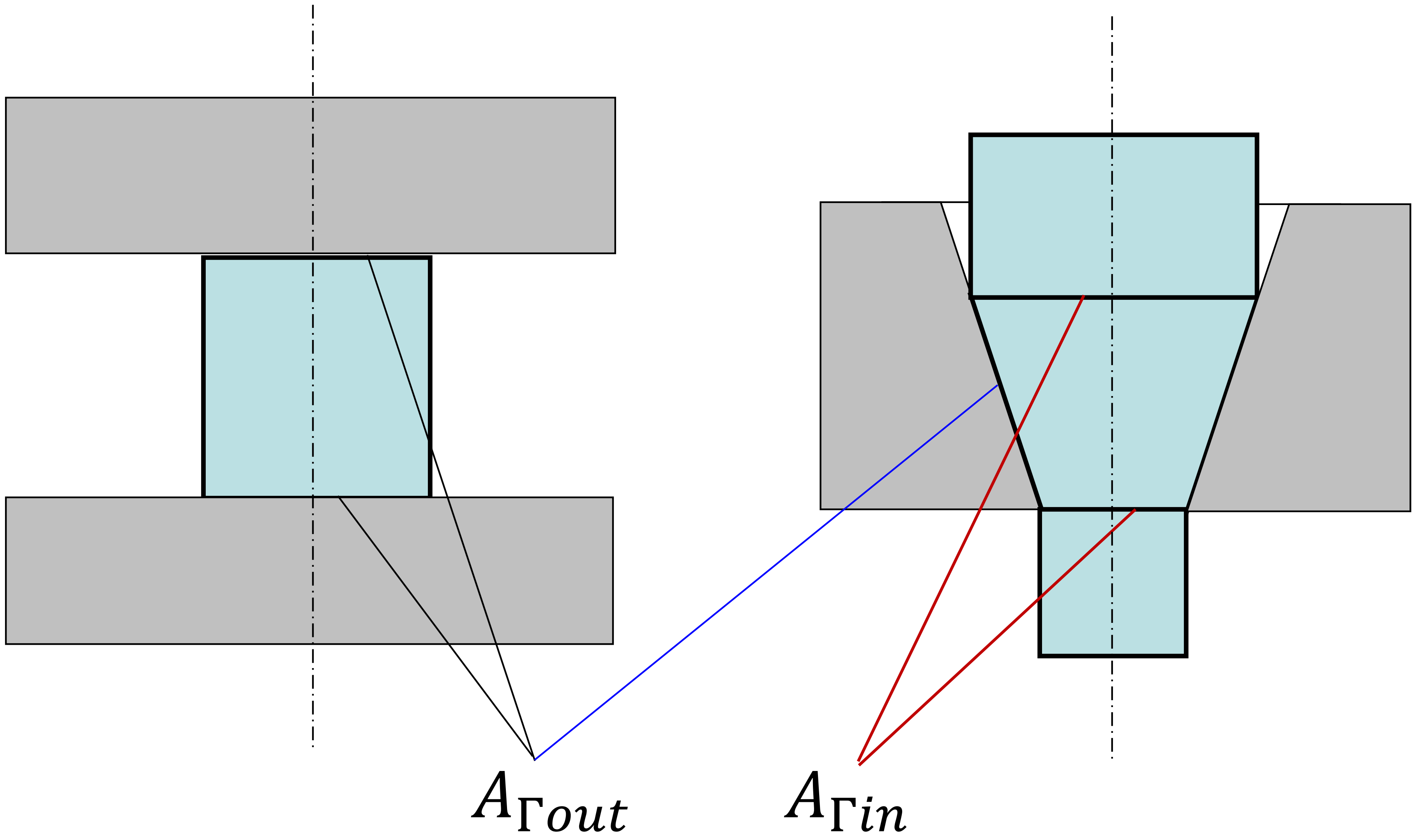


$$v_n^+ = v_n^-$$

$$\Delta v = v_\tau^+ - v_\tau^- \neq \mathbf{0}$$

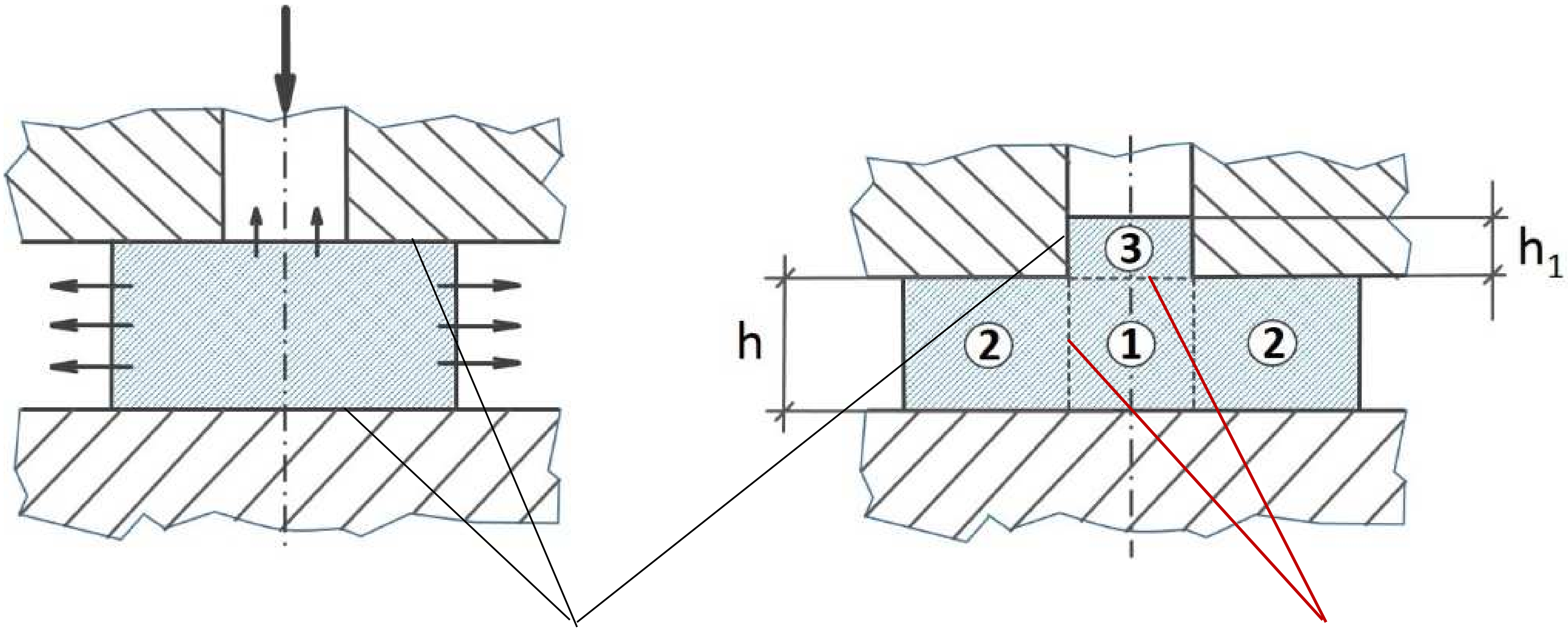
$$\Delta\bar{\varepsilon} = \frac{|\Delta v|}{\sqrt{3}(v_n - v_{ns})}; \qquad \bar{\sigma}_\Gamma = \frac{1}{\Delta\bar{\varepsilon}} \int_{\bar{\varepsilon}}^{\bar{\varepsilon}+\Delta\bar{\varepsilon}} \sigma_f \, d\bar{\varepsilon}$$

Discontinuity on boundaries



Discontinuity on boundaries

An application – the final shape is uncertain



$A_{\Gamma out}$ $A_{\Gamma in}$

$$J = J_1 + J_2 + J_3 = F(h_1) \Rightarrow \frac{\partial J}{\partial h_1} = 0 \Rightarrow h_1$$

Steps of energy approach

- Based on the actual flow, define all components of the velocity field.
- Defining the other components based on incompressibility (solution of a differential equation)
- Construction of kinematically possible strain rate fields
- Power of inner forces and discontinuity surfaces
- Finding the extrema of the functional.